EVALUATION OF THE KOMANDI TYPE SHEAR AREA EQUATION USING A TYRE TRACTION MODEL AND EXPERIMENTAL DATA

EVALUAREA RELAŢIEI DE TIP KOMANDI CU AJUTORUL MODELĂRII ŞI A DATELOR EXPERIMENTALE

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Abstract. *The paper tries to evaluate the goodness-of-fit of the shearing area equation developed by G. Komandi, using a traction model previously developed by the authors as well as experimental traction data. The traction model and the field tests referred to the driving tires of a Romanian U-650 tractor; comparison between experimental data concerning the traction force and contact area given by the traction model allowed evaluation of the shear area. As a result, a power type relationship (* $A_{sh} = A_t \cdot a \cdot s^b$, *where* A_{sh} *is the shear area,* A_t *is the contact area, a and b are constants and s is the wheel slip) was found to better describe the shear area instead of the power type one proposed by Komandi.*
Key words: tire traction, shear area, contact area

Rezumat. *Lucrarea încearcă să evalueze formula dezvoltată de G. Komandi, folosind un model pentru tracţiunea roţii motoare dezvoltat anterior de catre autori, precum şi date experimentale. Modelul şi datele experimentale se referă la tractorul U-650, iar aria zonei încărcate cu tensiuni tangenţiale a rezultat comparând rezultatele teoretice cu cele obţinute experimental.S-a constatat că o lege de variaţie de tip putere este mai potrivită în locul celei de tip exponenţial propuse de către G. Komandi.*
Cuvinte cheie: tracţiune, pneu, suprafaţă de contact, forfecare

INTRODUCTION

The traction of the wheel is a complex physical process and prediction of the traction performance of a tractor wheel depends largely on the model of the tire-terrain interaction. Some models use analytical approaches (commercial CAD or FEM programs), others use semi-empirical or empirical approaches (based on the model proposed by Bekker, Wong etc.) [1].

The semi-empirical method for traction prediction developed by Bekker assumes that the vertical deformation of the soil under the wheel load is similar to the one produced by a sinkage plate and that the shear deformation of the soil due to a traction device is similar to the shear action performed by a rectangular or torsional shear device. In this paper, the semi-empirical approach is used in order to evaluate the wheel-soil pressure and shear stress in soil; as shown in other papers [6, 7], the results given by this model are confirmed by experimental data. The values of the soil constants used by the model are obtained from plate penetration tests and shear stress-shear displacement tests.

In most of the developed traction motion models the shearing surface is considered to be constant. As Komandi has shown in [5], “The shearing surface varies while the tire develops the tractive force but, for practical purposes, shear stress does not change, except for a very small decrease which may occur after sliding begins. The shearing surface can vary from zero to the entire contact surface”. Taking these facts into account we decided to combine a previously developed model for predicting the shear stress and experimental traction data in order to establish whether the Komandi equation may be applied for our soil conditions, or, otherwise, to find a valid equation for the variable shearing surface.

MATERIAL AND METHOD

1. Traction model

The traction model is based on the schematics shown in Figure 1a. The model assumes that, under the vertical load (G), the wheel sinks into the soil, reaching depth (z_c) and the load induces tire deflection (z_p). As a result, the radius of the contact patch becomes r_d ($r_d > r_0$), and the circular length of the contact patch is:

$$l_c = 2 \cdot \sin\beta \cdot r_d = 2 \cdot \sin\alpha \cdot r_0 \,. \tag{1}$$

From Figure 1 we have:

$$z = O\overline{E} - O\overline{A} \tag{2}$$

and we finally get:

$$z = r_d \cdot \left[\cos(\beta - \varphi) - \cos\beta\right] \tag{3}$$

Using the Bekker equation $p = k \cdot z^n$ results in:

$$G = \int_0^{2\beta} p \cdot b(\varphi) \cdot r_d \cdot \cos(\beta - \varphi) \cdot d\varphi = \\ = k \cdot \int_0^{2\beta} r_d^{n+1} \cdot \left[\cos(\beta - \varphi) - \cos\beta\right]^n \cdot b(\varphi) \cdot \cos(\beta - \varphi) \cdot d\varphi, \tag{4}$$

where, according to Figure 1b:

$$b(\varphi) = \sqrt{\frac{l_c^2 \cdot l_w^2 - 4 \cdot l_w^2 \cdot r_d^2 \cdot \sin^2(\beta - \varphi)}{l_c^2}} \,. \tag{5}$$

Assuming the tire is perfectly elastic, we have [3]:

$$G = q_p \cdot b \cdot \frac{4}{3} \cdot \left(\alpha^3 \cdot r_0^2 - \beta^3 \cdot r_d^2\right) \tag{6}$$

where q_p is the volume deflection constant of the tire.

From (4) and (6) we get:

$$k \cdot \int_0^{2\beta} b(\varphi) \cdot r_d^{n+1} \cdot \left[\cos(\beta - \varphi) - \cos\beta\right]^n \cdot \cos(\beta - \varphi) \cdot d\varphi + \\ + \frac{4}{3} \cdot b \cdot q_p \cdot \beta^3 \cdot r_d^2 = \frac{4}{3} \cdot b \cdot q_p \cdot \alpha^3 \cdot r_0^2 \tag{7}$$

From Figure 1 we also have:

$$z_c = r_0 - z_p - r_0 \cdot \cos\beta \,, \tag{8}$$

$$z_p = r_0 \cdot (1 - \cos\alpha) - r_d \cdot (1 - \cos\beta) \,, \tag{9}$$

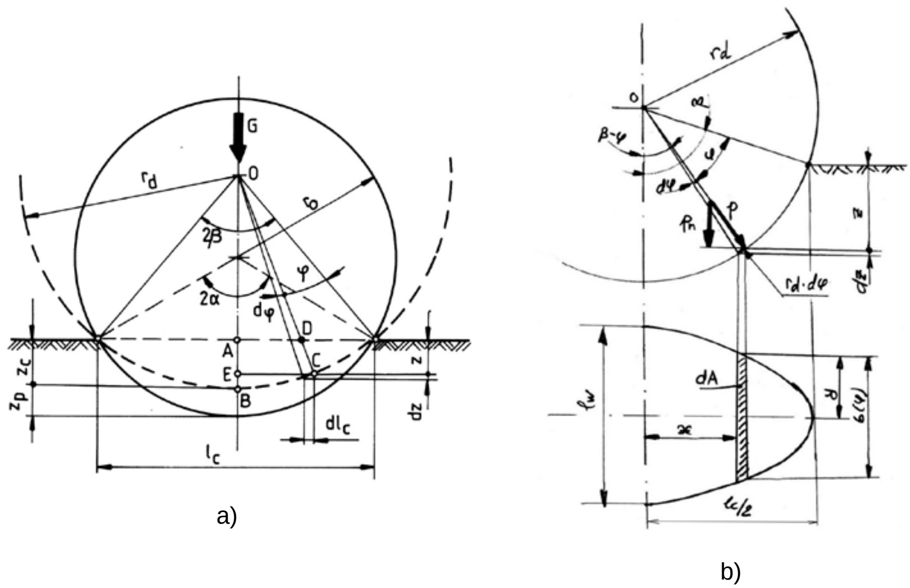

Fig. 1. Schematics of the model

The system consisting of equations (1), (7), (8) and (9) is solved with a computer program, using an iteration process, as shown in [7].

The contact patch is assumed to have an elliptical shape [12], with l_c the major axis and l_w the minor axis; the area of the contact patch is:

$$A_t = \frac{\pi}{4} \cdot l_c \cdot l_w \tag{10}$$

According to Komandi [5], the traction force is given by the sheared area and not by the overall area of the contact patch; moreover, the sheared area varies during the tractive activity exerted by the tire:

$$A_{sh} = A_t \cdot D(s) \, , F_t = \tau \cdot A_{sh} - R_r \tag{11}$$

where A_{sh} is the sheared area, s is the wheel slip, F_t is the traction force, τ is the shear stress on the tire-terrain interface and R_r is the wheel rolling resistance. The variable parameter D(s) is given by Komandi as a function of wheel slip:

$$D(s) = 1 - (1 - s) \cdot e^{-c_1 \cdot l_c^{m_1} \cdot s^{m_2}} \tag{12}$$

and the values of the constants c_1, m_1 and m_2 depend upon the nature of the surface.

2. Experimental setup

For this work the U-650 tractor was modeled. During the experiments, drive wheel slip and net traction force $F_{t,ef,r}$ were measured directly. The experimental data were collected during field tests of the U650+P2V ploughing unit (aiming to evaluate the quality of the plough's working process); during these tests drive wheel slip was not allowed to exceed 30% because such high values must be avoided during the ploughing process.

The test conditions are shown in Table 1.

The rolling resistance and shear stress were evaluated using the procedure presented in [6, 7].

As the traction force is known and shear stress, contact patch area and rolling resistance are given by the traction model, the D(s) term in equation (11) was calculated with the formula:

$$D(s) = \frac{F_{t,ef,r} + R_r}{\tau \cdot A_t} \qquad (13)$$

Table 1

Test conditions

Item		Value
Load on the drive tire [kN]		11.75
Type of drive tire		14.00 – 38
Overall diameter of tire [m]		1.58
Tire width [m]		0.367
Distance between lugs [m]		0.195
Transversal radius of the undertread [m]		0.3
Soil deformation modulus, K [m]		0.05
Coefficients for the sinkage equation	k	55
	n	1.3
Soil cohesion, c [kPa]		25
Angle of internal friction, φ [^{0}]		32
Cone penetrometer index, CI [kPa]		970

Then the software package LABFit [9] was used in order to evaluate the goodness of fit of a mathematical expression to the calculated values of the D(s) parameter, using the “Curve fit” subroutine.

RESULTS AND DISCUSSIONS

The results concerning the D(s) parameter, starting from experimental traction data and using the relation (13) are shown in Table 2.

Fitting the Komandi type relationship to the values of the D(s) parameter led to the results shown in Figure 2. The equation for the D(s) parameter is:

$$D(s) = 1 - (1 - s) \cdot e^{-1.349 \cdot (l_c)^{-1.393} \cdot s^{0.657}} .$$

Unfortunately, the coefficients involved in this equation register very high values of the standard deviation and, in the meantime, Figure 2 displays extended confidence and predict bands. As a result we concluded that, for the soil conditions taken into account, the Komandi type relationship does not fit very well to the experimental data.

Using the functions library of the LABFit software we tried to find a better fit to the experimental data. As a result we found a power type equation:

$$D(s) = a \cdot s^b ,$$

where: a = 1.4 ± 0.07877 and b = 0.449 ± 0.0335. The correlation coefficient for this set of data was r^2 = 0.96933 and the results are depicted in Figure 3, which displays much narrower confidence and predict bands compared to the Komandi type equation.

In order to compare the results given by the traction model and experimental data, in terms of traction force, the power type relation was used and the results are shown in Figure 4.

Table 2

Values for the D(s) parameter

Slip	D(s)
0.06	0.4065
0.09	0.4740
0.14	0.5990
0.17	0.5779
0.18	0.6330
0.20	0.7090
0.25	0.753
0.26	0.763
0.29	0.812

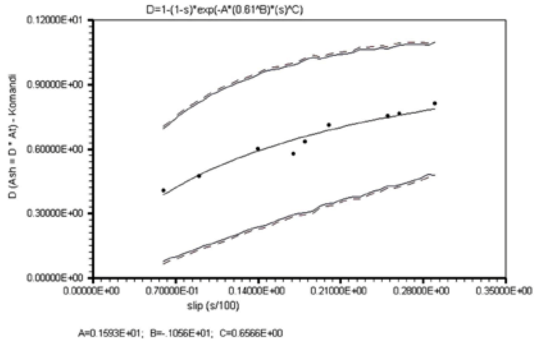

Fig. 2. Results concerning the Komandi type equation

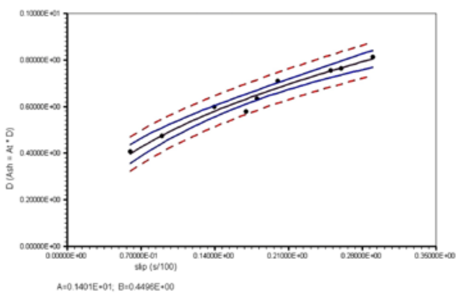

Fig. 3. Results referring to the power type equation

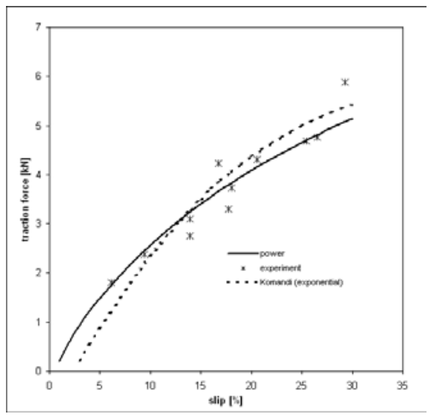

Fig. 4. Traction force

Using the percentage of points within 95% confidence interval of data (Pw95CI) as a measure of degree of fit of model to experimental data [8] we found that, for the power type equation of the D(s) parameter, 88.9% of the points fall within this interval, compared to only 44.4% when the Komandi equation is used.

The mean absolute deviation (MAD), representing the mean of the absolute value of deviation between each model prediction point and its corresponding data point, has a value of 0.2010 for the power type relation and respectively 0.3637 for the Komandi type equation.

Computation of the mean scaled absolute deviation (MSAD) [8] shows that, on average, the model is 1.193 standard errors off from the experimental data when the power type relationship is used, compared to MSAD = 2.323 for the Komandi relation.

CONCLUSIONS

A previously developed traction model and experimental data were used in order to calculate the traction force and shear area.

The shear area was supposed to be linked to the tire-ground contact area by the means of a wheel slip depending parameter D(s).

Using the LABFit software in order to calculate the coefficients involved in the D(s) relationship developed by G. Komandi led to conclusion that this equation did not seem to be appropriate for the evaluation of the shear area, at least not for our test conditions.

A much better goodness-of-fit between the measured traction forces and the ones predicted by our model was achieved when a power type relation was used in order to describe the D(s) parameter.

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